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Wojakowski, Rafal M.; Ebrahim, M. Shahid; Jaafar, Aziz; Osman Salleh, Murizah

### Financial Markets, Institutions and Instruments

DOI:

[10.1111/fmii.12109](https://doi.org/10.1111/fmii.12109)

Published: 31/05/2019

Peer reviewed version

[Cyswllt i'r cyhoeddiad / Link to publication](#)

*Dyfyniad o'r fersiwn a gyhoeddwyd / Citation for published version (APA):*

Wojakowski, R. M., Ebrahim, M. S., Jaafar, A., & Osman Salleh, M. (2019). Can a Loan Valuation Adjustment (LVA) Approach Immunize Collateralized Debt from Defaults? *Financial Markets, Institutions and Instruments*, 28(2), 141-158. <https://doi.org/10.1111/fmii.12109>

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# Can a Loan Valuation Adjustment (LVA) Approach Immunize Collateralized Debt from Defaults?<sup>1</sup>

Rafal M. Wojakowski<sup>2</sup>      M. Shahid Ebrahim<sup>3</sup>      Aziz Jaafar<sup>4</sup>

Murizah Osman Salleh<sup>5</sup>

September 17, 2018

<sup>1</sup>We are grateful to Fotios Pasiouras, the Editor and two anonymous referees for their suggestions which greatly helped us improving the paper. An earlier version of this paper titled as “On Immunizing Collateralized Debt from Defaults” was presented at the 2016 Portsmouth-Fordham Conference on Banking and Finance. We thank Kartik Anand, Iftekhar Hasan, Peter Jackson, Lesley Reinhorn and participants of various conferences for their critical comments on earlier drafts of this paper.

<sup>2</sup>*Corresponding Author:* Reader in Finance, University of Surrey, Faculty of Arts and Social Sciences, Surrey Business School, Guildford, GU2 7XH UK. Work phone: +44 1483 683477 e-mail: r.wojakowski@surrey.ac.uk

<sup>3</sup>Professor of Finance, Durham University Business School, Mill Hill Lane, Durham DH1 3LB, UK. Work phone: +44 1913 345405 e-mail: m.s.ebrahim@durham.ac.uk

<sup>4</sup>Professor of Accounting, Bangor University, Bangor Business School, Hen Goleg, Bangor, Gwynedd LL57 2DG, UK. Work phone: +44 1248 383226 e-mail: a.jaafar@bangor.ac.uk

<sup>5</sup>Deputy Director, Bank Negara Malaysia (Central Bank of Malaysia), Jalan Dato’ Onn, 50480 Kuala Lumpur, Malaysia. Work phone: +603 2698 8044 e-mail: mosmansalleh@yahoo.co.uk  
The views in the paper are solely the author’s and do not represent the Central Bank of Malaysia.

# Can a Loan Valuation Adjustment (LVA) Approach Immunize Collateralized Debt from Defaults?

## Abstract

This study focuses on structuring tangible asset backed loans to inhibit their endemic option to default. We adapt the pragmatic approach of a margin loan in the configuring of collateralized debt to yield a quasi-default-free facility. We link our practical method to the current [Basel III \(2017\)](#) regulatory framework. Our new concept of the Loan Valuation Adjustment (LVA) and novel method to minimize the LVA converts the risky loan into a quasi risk-free loan and achieves value maximization for the lending financial institution. As a result, entrepreneurial activities are promoted and economic growth invigorated. Information asymmetry, costly bailouts and resulting financial fragility are reduced while depositors are endowed with a safety net equivalent to deposit insurance but without the associated moral hazard between risk-averse lenders and borrowers.

**JEL codes:** D53, G10, G20, G28.

**Keywords:** collateral, loan default, financial innovation, financial regulation, financial fragility, agency cost.

*“Conflicts between debt and equity only arise when there is a risk of default. If debt is totally free of default risk, debtholders have no interest in the income, value or risk of the firm.”*

(Stewart C. Myers, 2001, p.96)

## 1 Introduction

The relevant features of a collateralized loan are the current value of its collateral and the remaining balance and thus the current exposure. The future value of the collateral is random and contingent on its prospective exposure. Risk intensifies when the market value of the collateral declines and the borrower defaults.

As time progresses, the default risk is subject to two effects. First, uncertainty attached to the value of the collateral. This becomes acute the closer the term, thereby exacerbating default. Second, loan contracts, such as corporate loans or mortgages secured on the value of real estate property, involve cash flows that are paid over time. This reduces the remaining balances as the underlying loans amortize through time. As a result, the *maximum exposure* is unlikely to occur in the first year.<sup>1</sup> The maximum exposure is also unlikely to be in the last years since most of the payments will already have been made by then. It is more likely that the maximum exposure will be in the middle of the tenure of the contract.

This paper proactively proposes a new method for tackling both risk shifting as well as underinvestment. This is conducted by quantifying *and modifying* the maximum exposure to default risk by looking forward into the payment schedule of a loan contract. We show that the probability of default can be minimized by making the initial loan-to-collateral value ratio small enough. Since the maximum exposure is in most cases located in the middle of the contract, a static model focusing on initial exposure or a specific default

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<sup>1</sup>For example, a 30-year mortgage normally requires an initial loan to value ratio such as 90% or lower and an instant decline in the value of the house by 10% or more during the first year is rather unlikely, at least in a non-turbulent economy.

(terminal) date assumed *ex-ante* is inappropriate for the task. An optimal level of debt simultaneously addresses the underinvestment problem for the borrower. Thus, we need a forward-looking, stochastic *and* intertemporal model to solve this issue. Consequently, we choose a framework which is capable of incorporating the impact of: (a) the probability of default; (b) the present value of the maximum lifelong exposure, on the initial permissible loan to value ratio; while (c) endowing financial flexibility to the borrower to expand his venture.

Unlike prevailing practice, we go beyond the current exposure at origination approach. We also include the maximum life-long exposure into our analysis. Therefore, our new method also integrates the potential future loss that may occur over the lifetime of a contract due to a borrower defaulting on her/his loan. We are proactive and contribute above the standard approach which normally only considers the current value of the collateral and/or a specific default date.

Our approach is inspired by the Credit Valuation Adjustment (CVA) concept which is now an integral part of the [Basel III \(2017\)](#) regulatory framework. CVA is the difference between the price of a default-free derivative and the price of a default-prone derivative, to account for the expected loss from counterparty default. Following the crisis of 2007, CVA adjustments are now required daily by [Basel III](#) for exposures to derivatives in the context of counterparty credit risk. What we essentially do is reverse the logic of CVA. In contrast, the CVA remains only a risk measuring tool.

Our enhanced framework can be applied to decrease (and, ideally, quasi-eliminate) the likelihood of default when approving the size of collateralized loans. We define 'Loan Valuation Adjustment' (*LVA*) as the difference between the value of a risk-free loan and the value of a risky loan, such that a default-prone loan has a lower value than a risk-free loan. This is because a borrower in a default-prone loan may renege on his/her obligations and the bank issuing such a facility will not receive the scheduled payments,

i.e. the amortized fraction of principal plus interest on remaining balance.

This paper incrementally contributes to the extant literature in four ways described below.

Our first contribution to the literature is to provide a method to minimize the *LVA* and thus to provide an answer to the problem of converting a risky loan into a quasi-risk-free loan. Unlike the *CVA*, which is exogenously driven by the state variables of the economy<sup>2</sup> and thus cannot be changed, our *LVA* can be proactively implemented and modified. Our approach looks forward into future points in time for the lifetime of the loan. We obtain potential future exposures at each future time point. We then aggregate positions backwards, taking into account values of the collateral and remaining balances. We then derive implications on the maximum allowable quasi-risk-free loan which can be granted today, and thus maximize its value to the lending financial intermediary institution.

Our second contribution is to offer a method which is capable of mitigating the twin issue of *risk-shifting* (illustrated in Figure 1) where the borrower defaults between periods  $t_1$  and  $t_2$  (when the equity is underwater) and *underinvestment* (illustrated in Figure 2) when the borrower's cash-flows in period 2 are below its debt obligations. Tackling these twin issues is crucial, as they impact on economic fragility, dampen entrepreneurial activity and thus economic growth. Our approach is linked to the concept of margin loans where: (i) the underlying asset is over-collateralized (to offset risk-shifting); and (ii) an optimal tenure of facility is evaluated to allow the borrower to meet his/her debt obligations (to offset underinvestment).

[Figures 1 and 2 about here]

Third, our results are in agreement with the theoretical adverse-selection screening

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<sup>2</sup>Quantifying *CVA* typically involves simulating risk factors at numerous future points in time for the lifetime of the derivatives' book, re-pricing positions at each time point, and aggregating positions on a path consistent basis, taking into account netting and collateral posted or received. In particular, there is no specific guidance on the methods used to calculate *CVA*. See [Chatterjee \(2015\)](#).

models of e.g. [Bester \(1985\)](#) or [Manove, Padilla, and Pagano \(2001\)](#), where good (less-risky) borrowers are required to post collateral and bad (more risky) borrowers select into loan contracts with no collateral but with e.g. higher interest rates. At first sight such theoretical results may seem inconsistent with much empirical work (see for example [Berger and Udell, 1990](#)), who find that collateral is required from high-risk borrowers. However, they are not. If banks systematically apply methods (such as proposed in this paper) to screen out risky borrowers, only borrowers able to post enough collateral would obtain funding. Riskier projects would need more collateral or would never be launched. However, entrepreneurs with less or no collateral would be able to fund and implement less risky projects instead. As a consequence, if one were to use ex-post performance to partition borrowers into low risk and high risk classes, one would conclude that collateral is posted by the high-risk borrowers.

Finally, our fourth contribution is to link our framework to the problem of pragmatically eliminating agency issues from debt, where the loan is collateralized with tangible (i.e., real) assets. Our proposal is consistent with [Myers \(2001\)](#), who infers the efficiency of default-free collateralized loan structure as mitigating agency cost. We: (i) illustrate that such a structure warrants a treatment different from that of information asymmetry; (ii) extend the scarce literature on pricing collateralized loans devoid of agency costs of debt; and (iii) introduce a novel margin approach to collateralized loan pricing, which provides a way to reduce the fragility of the financial system while endowing financial maneuverability to the borrower.

Prudent underwriting warrants satisfaction of both (i) asset; and (ii) income constraints to thwart the risk-shifting and underinvestment issues, respectively. Our loan pricing mechanism is consistent with [Baltensperger \(1978\)](#) who advocates incorporation of not only the interest rate but also the loan-to-value ratio and the tenure of the facility. [Archer and Smith \(2013\)](#), and [Foote, Gerardi, and Willen \(2008\)](#) in their studies extend

the parameters by including borrower income factors.<sup>3</sup> This satisfies a higher order risk management approach in contrast to ad hoc credit rationing practices, and overall loan loss rehabilitation programs (see again [Ambrose and Buttimer, 2000](#); [Foote et al., 2008](#)).

Our analysis is from the perspective of a (foresightful) lender, who makes allowances for the borrower to avoid succumbing to underinvestment. This implies that the lender makes the prudent assumption that, should the borrower default, it will be extremely unlikely that the borrower will adequately and timely recover the shortfall outstanding after the surrendered collateral is auctioned. Furthermore, in order to make our exposition simple, in our set-up we assume a non-recourse bankruptcy regime (like in the US) and lending against tradable collateral (e.g. a house, like in mortgage lending). However, in many countries outside the US, personal bankruptcy legislation is very stringent and surrendering the collateral to the lender does not allow the borrower to escape his debt obligations. Furthermore, independently of the recourse (or non-recourse) nature of the loan regime which affects borrowers, legislation in some countries tend to protect creditors less. As a result in countries such as France, lenders may anticipate this and require more collateral or increase the contract interest rate; see [Davydenko and Franks \(2008\)](#). Our analysis can thus be expanded to include these different extremities on the lender and borrower sides. (See also footnote 4.)

Mathematically, this paper employs a continuous time setup to follow a sequence of steps, involving amortizing collateralized debt. However, unlike [Ebrahim \(2009\)](#), who works in discrete time in the context of housing finance cooperative, we work in continuous time and we derive our contributions from the concept of margin lending in our novel framework of Loan Value Adjustment. Our paper can thus be seen as an extension of the [Ebrahim \(2009\)](#) analysis (involving home mortgages) to a more rigorous and robust

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<sup>3</sup>[Archer and Smith \(2013\)](#), and [Foote et al. \(2008\)](#) observe that the debt-to-income ratio and unemployment rate positively influence the put option to default.



one and based on option pricing technology (involving a broader corporate sector of the economy). Our analysis is thus improved and our framework superior.

Our analysis is related to [Jokivuolle and Puera \(2003\)](#) who also focus on collateral value. However, the central issue we aim at in our paper is the exposure pattern *through time*. Although in their derivations they also use continuous-time stochastic processes, their model is *de facto* static, with a unique exposure point. This is because they assume a single possible default date, which must be known *ex ante* and which in their model is set the same as the maturity of the debt. This makes their analysis valid only for zero-coupon, no-repayment case and is counterfactual because defaults are more frequent in early stages of loans' life-cycles. In contrast, our model is intertemporal as we examine the continuum of future exposures, which is the cornerstone of our approach to derive the LVA. Moreover, our model is more realistic and versatile as it naturally covers the more prevalent amortizing case, where default can occur at any time when principal repayment and interest are due before maturity.

Our analysis is also related to [Frontczak and Rostek \(2015\)](#) who, unlike our efforts to provide a dynamic risk measure (LVA), focus on mortgage backed loans and static point measures such as Loss Given Default (LGD). More importantly, [Frontczak and Rostek \(2015\)](#) note that while [Jokivuolle and Puera \(2003\)](#) use correlated processes for the firm value on one hand and asset value on the other hand, their formulas "lack of concreteness towards realistic values of collateral." Put differently, and as already noted by [Jokivuolle and Puera \(2003\)](#) themselves, their formulas are technically correct only when the debtor does not own the collateral. Mathematically, this problem arises because a sum of two correlated geometric Brownian motions is not a geometric Brownian motion. Similar difficulties also arise in pricing payoffs which depend on arithmetic price averages, such as Asian options, see [Geman and Yor \(1993\)](#), and approximations must be used, see [Chung, Shackleton, and Wojakowski \(2003\)](#). In contrast, our focus is on the exact opposite case,

i.e. when the debtor owns the collateral. This is also the setup adopted by [Frontczak and Rostek \(2015\)](#) who, like us, use a single stochastic process and an amortizing outstanding principal schedule. An interesting feature of the [Frontczak and Rostek \(2015\)](#) model is, as advocated by [Fabozzi, Shiller, and Tunaru \(2012\)](#), to employ an exponential Ornstein-Uhlenbeck process (instead of the most commonly used geometric Brownian motion) to reflect the incompleteness of real estate market, seasonality and significant serial auto-correlations observed in house prices. While we focus on a wider range of loans than mortgages, an obvious extension of our results would be to use the exponential Ornstein-Uhlenbeck process and re-derive our closed form formulas when specializing *LVA* to mortgage backed debt.

The implications of our results are very important and are described as follows. The reverberation of the subprime crisis has led regulators to impose higher capital requirements on banks. Though this moderates moral hazard, it may not be sufficient if banks ignore to price their loan facilities meticulously. This would then result in handing out to borrowers an in-the-money strategic option to default. We argue that regulators should compel banks to price their debt facilities in a way that divests the put option to default thereby sterilizing the feedback loop connecting collateral and credit cycles. This would consequently diminish the funding constraints on borrowers to alleviate the underinvestment issue. This would eventually strengthen the financial sector, augment its resiliency and boost entrepreneurial activity in the economy.

This paper is structured as follows. The next section, [2](#), discusses the relationship of collateral debt to adverse selection and moral hazard agency issues. Section [3](#) discusses the concept behind margin loans, our Loan Valuation Adjustment (*LVA*) framework and their relationship to collateralized debt. The subsequent Sections discuss our formal method of structuring collateralized loans to free them from risk of default as much as possible (Section [4](#)) and constraints required, the asset value constraint (Section

5) and the income constraint (Section 6). The final section, 7, concludes.

## 2 Loan Valuation Adjustment (LVA) framework v.s. agency theory and behavioural finance

Studies on security design should delve into agency theory or/and behavioural finance as these are now mainstream. With respect to agency theory we position our paper as follows. Agency theory and our framework reflect the same or similar effects. That is, risk shifting and underinvestment. In our setup these are induced by a mechanism different from information asymmetry. However, while still naming these effects “agency issues,” we argue that risk shifting and underinvestment arise, respectively, when borrowers have no or not enough “skin in the game” or/and high Debt Service to Income (DSI) ratio. In what follows we elaborate on the main differences.

The economics literature defines agency issues as emanating from a conflict of interest between a principal and an agent, where one party has an informational advantage over the other (see [Leland and Pyle, 1977](#)). Asymmetric information in a financial contract is exemplified when a lender (principal) or a borrower (agent) possesses greater knowledge than the other. This situation can manifest itself in two cases. Information asymmetry ex-ante (i.e., prior) or ex-post (i.e., after) entering into a financial contract. These are classified as adverse selection and moral hazard, respectively (see [Grossman and Hart, 1983](#); [Stiglitz and Weiss, 1981](#)).

[Holmstrom \(1979\)](#) explores the effect of principal-agent relationship in optimal debt contracts. However, his solution suggests the use of information systems, which reduce asymmetry with respect to the agent’s conduct. Although imperfect, it can improve the contract economic efficiency, i.e., presumably by treating information asymmetry it abolishes agency issues. Other measures to curb agency costs of debt include (i) call provisions that moderate information asymmetry and asset substitution issues; (ii) conversion

rights to curb management excessive consumption; and (iii) income bonds that alleviate bankruptcy problems (see [Myers, 1977](#)).

In the special situation of loans backed by tangible assets focused in this paper, we mitigate both adverse selection and moral hazard as follows:

1. The adverse selection issue is resolved by a financier by transferring funds to the buyer of the tangible asset after conducting an elaborate due diligence process where the title of the asset, its structural soundness and its value are verified. Furthermore, the financier can estimate the ex-ante probability distribution of actual (or imputed) payoffs from its ex-post risk-return information. This is accomplished by trading financial claims over a multi-period horizon ([Hosios and Peters, 1989](#)).
2. The moral hazard issue is resolved by ensuring that the title of the tangible asset has a lien on it by the financier to restrain the borrower from selling it. Furthermore, the financier can add iron-clad covenants to the loan contract to preserve the value of the underlying collateral. These include: (i) adequate maintenance of the real asset; (ii) payment of local taxes; and (iii) minimum insurance coverage ([Smith, Jr. and Warner, 1979](#)).

We reiterate the fact that agency issues in our collateralized financial contract do not ensue from asymmetric information. They ensue from low “skin in the game” or/and high Debt Service to Income (DSI) ratio (see again our Figures [1](#) and [2](#)).

As far as behavioural finance is concerned we note that irrational behaviour of agents, and in particular that of lenders, would be an extension of our framework worth considering. However, like in most studies in financial economics we assume that agents are rational here. Under asymmetric information the borrower and the lender might have divergent expectations about the future profitability of the asset. One might think of a case where the lender might have an idea of how the borrower would react when forced

by her to default, irrespective of the information he has. However, such logic is incorrect if agents are rational. A rational lender would never make the borrower default in the interim period  $t_1$  to  $t_2$  (see Figure 1) since equity is negative in this situation and the lender would suffer a loss of principal. This would imply that the lender is irrational.

Consistent with the agency perspective of Allen (2001), we treat bankruptcy as arising from *endogenous* conflict of interest. Building on this result, we structure an iron-clad asset backed financial facility that moderates the agency issues. The goal is to pragmatically price the collateralized loan such that it simultaneously reduces the: (i) in-moneyness of the embedded put option to default; (ii) negative effect of the feedback loop between collateral and credit cycles; and (iii) financial constraints on borrowers to invest in the real sector of the economy.

We note, however, that there may be special cases of agency issues stemming from risk-shifting (related e.g. to strategic default: see Guiso, Sapienza, and Zingales, 2013) and underinvestment (related e.g. to predatory loans: see Bond, Musto, and Yilmaz, 2009) that do not conform to our framework. In these special cases agency issues cannot be fully purged from the facility (see also Jensen and Meckling, 1976; Myers, 1977).

### 3 Margin Lending, Loan Valuation Adjustment (LVA) and Collateralized Loans

Margin lending facilities offered by prime brokers rose to popularity when stock ownership became more common in the beginning of the 20th century. The idea was to help finance the purchase of the stock by supplying only a fraction (or margin) of its value, whilst the balance was funded as a loan by the broker. In good states, the borrower benefited from the high returns on amount invested. For example, from the investor's perspective, the value  $x$  of a 20% margin would grow by 100% (i.e. *double*) to  $2x$  if the stock rose in value by just 20%. On the other hand, a moderate fall of 20% would entirely wipe out the initial investment  $x$ . For falls below 20% the investor could retain the stock,

but would receive a *margin call* to deposit additional cash to meet the shortfall.

Likewise, in the case of collateralized loans, the underlying asset (collateral), whose initial value is  $P_0$ , is the analogue of the stock in margin lending. There are no margin calls. But the initial deposit  $ID$  (or *equity*, see below) is wiped out leaving the borrower to face a dilemma. This is on whether to continue servicing the debt, whose initial value is  $Q_0$ ; or to exit the asset market by surrendering the same to the lender. Subsequent market values of collateral are random while the debt is often amortizing to zero over horizon  $T$ . A default can occur at time  $0 < \tau < T$  when it is either triggered exogenously or when the better alternative is to surrender the asset to the lender. In the cases provided in the loan covenant, the lender can subject the borrower to a technical default. The lender then has the right to take over and dispose the collateral as a protection against the borrower for not responding to the margin call or non-repayment or default on the debt.

In the above setup the borrower can be seen as having an equity stake (whose initial value is equal to the initial deposit  $ID$ ) to acquire the collateral. In particular, this equity can be seen as a call option on the value of the collateral,  $P_t$ , where the strike price is equal to the remaining debt balance,  $Q_t$ . When the value of the collateral is below the value of the remaining balance, i.e.  $P_t < Q_t$ , this call option is out of the money. However, if this happens the borrowers have an incentive to default on their obligations. This can be interpreted as exercising their put option to default which is embedded in their contract. The collateral is the underlying asset for this put option and its strike price is also equal to the remaining debt balance,  $Q_t$ . Typically, the lender adds a risk premium to the interest rate on the loan contract, to progressively recoup the cost of this default put option as the debt is being repaid. However, if the decrease in the value of the collateral is quick and sudden, this exposes the lender to the risk of the default put being exercised.<sup>4</sup>

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<sup>4</sup>In the case of amortizing debt considered here the default put can only be exercised prematurely as it's value converges to zero at maturity  $T$  as the strike  $Q_t$  decreases to zero.

Our Figures 1, 3 and 4 exemplify a project (venture) with initial value  $P_0$  funded by trading financial claims on the project payoffs with a financial intermediary. To secure financing, entrepreneur-manager places an initial deposit,  $ID$ , that is further secured by the project payoffs. Hence, the amount financed  $Q_0$  is the difference between the two

$$Q_0 = P_0 - ID . \quad (1)$$

The initial deposit acts as a commitment device enforced on the borrower, which (i) locks in up-front equity to the lender; and (ii) reduces the borrower's risk shifting behavior.

**[Insert Figures 3 & 4 about here]**

Alternatively, we can represent the lender's exposure using our Loan Valuation Adjustment (*LVA*) framework as the difference between the value of a risk-free loan and the value of a risky (default-prone) loan

$$V = V_f - LVA , \quad (2)$$

where the default-prone loan,  $V$ , has a lower value than a risk-free loan,  $V_f$ . As demonstrated with the default put concept, the borrower in the default-prone loan has strong incentives to default on her/his obligations, when the value of the collateral becomes less than the remaining balance ( $P_t < Q_t$ ). As soon as this happens, the financial institution (FI) underwriting such loans will not receive scheduled payments; i.e., they will miss the amortized fraction of principal plus interest on remaining balance.

We argue that *LVA* should be actively acted upon and adjusted. That is, FIs should minimize the *LVA* and thus they should convert the risky loan into a quasi-risk-free loan,

so that

$$\max V \iff \min LVA \iff LVA \rightarrow 0 \implies V \rightarrow V_f \quad (3)$$

That is, loan value maximization for the bank can be achieved by minimizing the *LVA*. This occurs when the *LVA* tends to zero which raises the loan level to the risk free value and thus transforms the default-prone loan  $V$  into a (quasi) risk-free (i.e., default-free) loan  $V_f$ . We define *LVA*, similarly to CVA,<sup>5</sup> as the present value of the  $Q$ -expectation of the non-recovered losses resulting from borrower's default

$$LVA = E^Q \left[ (1 - R) \mathbf{1}_{\{\tau \leq T\}} e^{-r\tau} (Q_\tau - P_\tau)^+ \right] \quad (4)$$

where  $R$  is the recovery rate,  $E^Q$  denotes the expectation operator under the risk-neutral measure,  $\tau$  is the default time and  $\mathbf{1}_{\{\tau \leq T\}}$  is the default indicator.

#### 4 Structuring Collateralized Loans Free of the Endemic Risk of Default

How then do we configure a pragmatically default-free collateralized loan structure that ensures the put option to default is probabilistically negligible? This has major implications on financial system resiliency considering the negative experience of collateralized lending in the recent financial crisis and repercussions on lender of last resort policies.<sup>6</sup> We look forward into future points in time for the lifetime of the loan. We obtain potential exposures at each future time point. We then aggregate positions backwards, taking into account values of the collateral and remaining balances. We then derive implications

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<sup>5</sup>See Brigo and Masetti (2005) and Basel III (2017).

<sup>6</sup>Moe (2012) documents the recent shift in the traditional role of Central Banks from a lender of last resort to market maker of last resort and, lately, as a quasi-fiscal agent with respect to quantitative easing policies. Measures to stabilize the financial system have also impacted widening Central Banks' balance sheet due to the more accommodative collateral policies in exchange for its liquidity funding.



on the maximum allowable quasi-risk-free loan which can be granted today, and thus maximize its value to the lending financial intermediary institution.

There are divergent perspectives on modeling default in a debt contract in the literature (see Black and Scholes, 1973; Duffie and Singleton, 1999; Jarrow and Turnbull, 1995; Merton, 1974). These views are given by: (i) the *default intensity approach* of Jarrow and Turnbull (1995), with exogenous default; or the *structural approach* of Merton (1974), with endogenous default.

Instead of focusing upon the mechanism triggering default, our approach is to ascertain: (i) the put option to default will never be significantly in-the-money (to avert risk-shifting); and (ii) the borrower has sufficient cash flows to service his/her debt obligations throughout the tenure of the facility (to avert underinvestment). That is, we impose conditions under which  $P_t$  is unlikely to descend below the remaining balance  $Q_t$  for all  $t \in [0, T]$ . This renders the probability of having  $Q_t < P_t$  close to 100% at all times over the lifetime of the loan  $[0, T]$ . This is achieved by carefully pricing the endogenous loan parameters: initial loan value  $Q_0$ , initial deposit  $ID$  and loan tenure  $T$ ; given the underlying exogenous factors: initial asset price  $P_0$ , its mean  $\mu$  and volatility  $\sigma$ , safety margin  $\alpha$ , the borrower's income  $\theta$  and income multiplier  $b$ . The above parameters and our methodology are elaborated in the sections below. As a result this reduces  $LVA$  close to zero. This ensures that the : (i) borrower does not convey the project risk to the lender; and (ii) lender does not restrain the borrower's entrepreneurial activities.

The remaining balance of a self-amortizing loan can be stated as

$$Q_t = Q_0 \frac{1 - \exp \{-r(T - t)\}}{1 - \exp \{-rT\}}. \quad (5)$$

where  $Q_0$  is the amount financed and  $r$  signifies the *lender cost of funds*. For tractability,

asset prices are assumed to follow a Geometric Brownian Motion.<sup>7</sup> Thus, asset price at time  $t$  is given by

$$P_t = P_0 \exp \left\{ \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right\} , \quad (6)$$

where  $\mu$  and  $\sigma$  capture the mean and standard deviation of the period (annual) appreciation (or depreciation, if  $\mu < 0$ ) of the asset, respectively, while  $W_t$  is a standard Brownian Motion process under the “real world” probability measure.

Figure 1 presents a stylized situation where the entrepreneur-manager equity (project payoff relative to outstanding loan) is ‘underwater’ in period  $t_1$  to  $t_2$ , i.e.  $P_t < Q_t$ . While it is understood that borrowers with positive equity will never default, not all borrowers whose equity is underwater will automatically default. Actual default occurs where the equity is significantly underwater. That is, the borrower’s expectation of future asset prices and non-pecuniary costs (e.g. income capability, effect on the borrower’s credit score) substantially exceeds the benefits of continuing with the loan repayments (see [Ambrose and Buttimer, 2000](#); [Archer and Smith, 2013](#); [Foote et al., 2008](#)).

## 5 Asset Value Constraint to Alleviate Risk Shifting

In contrast to its default-prone counterpart, a pragmatically default-free asset is characterized by a safety margin ( $\alpha$ ) that ensures the equity to loan ratio is sterilized from asset market volatilities across states of the economy with very high probability. This is illustrated in Figures 3 and 4 for asset market with upward and downward trend, respectively. This safety margin should be pre-conditioned to accommodate both an upward or down-

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<sup>7</sup>We intentionally simplify exposition to the necessary minimum to preserve clarity in linking our mathematical model to the current [Basel III \(2017\)](#) regulatory framework. As a consequence of this simplification the economic content of our model may appear to be minimal. However, our framework can easily be extended to incorporate many more realistic features, for example: (a) jumps in asset prices by adding Poisson jumps to the diffusion as discussed in [Merton \(1976\)](#); (b) stochastic debt; etc.

ward trend in the asset price (see again Figures 3 and 4). Moreover, the degree of safety margin required is contingent on specificities of the underlying project/asset riskiness.<sup>8,9</sup> These obscurities call for stricter safety margin due to the innate difficulties in pricing such instruments that is consistent with the asset risk weight.

For a debt facility to be *nearly default free* we require that the asset value  $P_t$  should be “sufficient” to pay off the outstanding loan balance  $Q_t$ . More precisely, we require that the probability, at all times, is very high, such that the balance  $Q_t$  is lower or at most equal to the asset price  $P_t$ , reduced by a safety margin

$$P_t^* = P_t \exp \left\{ -\alpha \sigma \sqrt{t} \right\} , \quad (7)$$

where the safety margin over time is measured by a multiple  $\alpha$  of the underlying asset “riskiness”  $\sigma \sqrt{t}$ . That is, we require that the minimal such probability over the lifespan of the loan  $t \in [0, T]$  is at least equal to  $\frac{y}{100}$ , where  $y$  is a very high *confidence level* such as 95 or 99

$$\min_{t \in [0, T]} \text{Probability} (P_t^* \geq Q_t) \geq \frac{y}{100} . \quad (8)$$

In other words the *maximum tail probability of risk-shifting over the life span of the project*<sup>10</sup> is

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<sup>8</sup>The asset riskiness is influenced by its redeployment value (Benmelech and Bergman, 2011), transaction/dissipative costs (Boot, Thakor, and Udell, 1991; Jokivuolle and Puera, 2003; Lacker, 2001), asset drift, volatility and correlation with other firm assets (Jokivuolle and Puera, 2003). The duration gap between the loan default and repossession time of the collateral could also be accounted for in assessing the asset value.

<sup>9</sup>See (Coval, Jurek, and Stafford, 2009) for imprecision effects in: (i) pricing of structured products; and (ii) estimation of their default risk. Rajan, Seru, and Vig (2010) link this to a Lucas critique failure to appropriately recognize the behavior of the agent (borrower) and impact on the underlying asset (collateral).

<sup>10</sup>This can be conceptualized as the lifetime probability of the worst case scenario.

lower than

$$\max_{t \in [0, T]} \text{Probability} (P_t^* < Q_t) < 1 - \frac{y}{100} . \quad (9)$$

That is, the tail probability of risk is essentially capped by  $1 - \frac{y}{100}$ , which in practice is a very small number, e.g. 1%.

For strictly positive  $P_t^*$  and  $Q_t$  we have the equivalency

$$P_t^* \geq Q_t \iff \ln P_t^* \geq \ln Q_t . \quad (10)$$

We can thus rewrite the probability which appears in equation (8) as

$$\min_{t \in [0, T]} \text{Probability} \left( \ln P_t - \ln Q_t \geq \alpha \sigma \sqrt{t} \right) \geq \frac{y}{100} . \quad (11)$$

In the above equation, we have used the definition (7) to substitute  $P_t^*$ . Thus, the log function of the *net* equity should be probabilistically greater or equal to this prudent buffer,  $\alpha \sigma \sqrt{t}$ , with high enough probability i.e. above  $\frac{y}{100}$  for all  $t \in [0, T]$ . The left hand side of the above equation (11) can be simplified as follows

$$\ln P_t - \ln Q_t \geq \alpha \sigma \sqrt{t} \iff \ln \frac{P_0}{Q_t} + \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma W_t - \alpha \sigma \sqrt{t} \geq 0 . \quad (12)$$

In the above equation, we have replaced  $P_t$  with its definition (6). Defining the  $d_2(t)$  function of time  $t$ , akin to the  $d_2$  parameter which appears in the [Black and Scholes \(1973\)](#) formula

$$d_2(t) = \frac{\ln \frac{P_0}{Q_t} + \left( \mu - \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}} , \quad (13)$$

we can rewrite the right hand side of (12) simply as

$$-\frac{W_t}{\sqrt{t}} \leq d_2(t) - \alpha . \quad (14)$$

Since  $-\frac{W_t}{\sqrt{t}}$  is a random variable with the standard normal distribution, the probability of staying financially sound at time  $t$ , which appears in condition (8), can be computed explicitly as

$$\text{Probability}(P_t^* \geq Q_t) = \Phi(d_2(t) - \alpha) , \quad (15)$$

where  $\Phi$  is the cumulative standard normal distribution function and the  $d_2(t)$  function is defined in equation (13). The notable difference with the  $d_2$  parameter of the [Black and Scholes \(1973\)](#) formula, is that the strike of the default put option modelled here is not constant in time but is decreasing to zero as maturity  $t$  increases. Intuitively, this decreasing strike for maturity  $t$  is equal to the remaining balance,  $Q_t$ , defined by Equation (5), which tends to zero at the tenure  $T$  of the facility. Finally, note the linear dependence on the margin adjustment parameter  $\alpha$  in the argument of the cumulative probability. This yields a very simple and instinctive implementation scheme: (a) compute  $d_2(t)$ ; (b) subtract the sought level of dispersion multiple  $\alpha$ .

Since  $\Phi$  is a strictly increasing function and  $\alpha$  is an exogenously imposed constant, therefore the minimization problem (11) can be rewritten as a Kuhn-Tucker problem as follows

$$\min_{t \in [0, T]} d_2(t) \quad (16)$$

$$\text{s.t. } \Phi(d_2(t) - \alpha) \geq \frac{y}{100} . \quad (17)$$

Let  $\alpha_y$  be a *confidence threshold* such that

$$\alpha_y : \Phi(\alpha_y) = P(z \leq \alpha_y) = \frac{y}{100} ,$$

where  $z \sim N(0,1)$ . For  $y = 99$  the  $\alpha_y$  equals 1.645. Therefore, since  $\Phi$  is strictly increasing, condition (17) occurs if and only if  $d_2(t) - \alpha \geq \alpha_y$ . Minimization (16) can now be represented as

$$\min_{t \in [0, T]} d_2(t) \tag{18}$$

$$\text{s.t. } d_2(t) \geq \alpha + \alpha_y . \tag{19}$$

Two situations can occur:

1. The first situation occurs if condition (19) is *slack* and there is an *interior minimum*, given by the following first order conditions

$$\begin{cases} d_2'(t^*) = 0 \\ d_2(t^*) > \alpha + \alpha_y \end{cases} \tag{20}$$

Solving (20) requires a numerical approach.

2. Otherwise, condition (19) is *binding* and there is a *corner solution* given by

$$d_2(t^*) = \alpha + \alpha_y . \tag{21}$$

Likewise, solving (21) requires a numerical approach.

After numerically solving either (20) or (21) for  $t^*$ , the minimal dollar value of the initial deposit,  $ID_{\min}$ , can be obtained by multiplying the minimal initial deposit fraction,

$1 - l_y(t^*)$ , by the initial project value  $P_0$ . This is explained in the numerical Example 1 below and illustrated in Figure 5.

[Insert Figure 5 about here]

**Example 1** For our base case parameters  $\{\mu, r, \sigma, T\} = \{0.1, 0.05, 0.2, 30\}$  the function  $d_2(t)$  and the threshold  $\alpha + \alpha_y = 2.645$  (chosen so that  $\alpha = 1$  and  $y = 99$ ) are illustrated on Figure 5. It is easily seen that in this case

$$\lim_{t \rightarrow 0} d_2(t) = 0 \quad \lim_{t \rightarrow \infty} d_2 = +\infty \quad (22)$$

and the function  $d_2(t)$  is strictly increasing over  $[0, T]$ . As  $0 < (\alpha + \alpha_y) < +\infty$ , the constraint (19) is therefore binding and there is a corner solution obtained by numerically solving (21), equal to  $t^* = 20.8312$ . Consequently, the prudent buffer is set at  $\alpha\sigma\sqrt{t^*} = 0.2\sqrt{20.8312} = 0.91282$ . Without optimization (11) over  $t \in [0, T]$ , the maximal loan value would be capped at the underlying asset value at point  $t = 0$ , that is  $Q_0 \leq P_0$ . This would lead to a loan-to-value ratio of 100%, which exacerbates moral hazard and thus financial fragility. With the help of optimization the maximal loan amount (for an asset with an upward trend) is thus reduced and the loan to value capped at

$$Q_0^{\max} = P_0 e^{-\alpha\sigma\sqrt{t^*}} \quad (23)$$

$$\implies l_0^{\max} = l_y(t^*) = e^{-\alpha\sigma\sqrt{t^*}} = e^{-0.91282} = 0.40139 \quad (24)$$

to ensure some ‘skin in the game.’ This yields the minimal initial deposit (or ‘skin in the game’) as a function of the underwriting asset value constraint along with the characteristics of assets

$$ID_{\min} = P_0 (1 - l_0^{\max}) = P_0 (1 - 0.40139) = 0.59861 P_0 . \quad (25)$$

It should be noted that when an asset follows an upward trend, initial loan disbursed by the financial intermediary yields a corner solution. In contrast, an asset following a downward trend yields an interior solution (see Figure 4). This is evaluated by solving for the optimal  $t^*$  in  $Q_{t^*} = P_{t^*}e^{-\alpha\sigma\sqrt{t^*}}$ . We then work backwards to evaluate  $Q_0$  given its relationship with  $Q_{t^*}$  and affordability, which is represented by the maximal periodic payment level ( $a_{\max}$ ). This is discussed in the next Section, 6, where we also extend our analysis to determine the optimal tenure of the lending facility.

## 6 Income Constraint to Alleviate Underinvestment

The repayability of a fully amortizing loan is contingent on (i) periodic annuitized payments  $A$ ; as well as (ii) income capability  $\theta$  of the borrower (i.e., firm).<sup>11</sup> Define a continuous coupon rate  $a$  (analogous to the discrete-time  $A$ ), for example as

$$e^a = (1 + A)^{12} \implies a = 12 \ln(1 + A) , \quad (26)$$

if  $A$  is the corresponding monthly coupon. Prudent underwriting warrants the periodic payments to be within the borrowing firm's income capability

$$\frac{\theta}{a} \geq b , \quad (27)$$

where  $b$  is the income multiplier. By rearranging this rule, one can restate the maximal periodic payment as

$$a_{\max} = \frac{\theta}{b} . \quad (28)$$

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<sup>11</sup>The assumption of a fully amortizing loan is for illustrative purposes. In our conceptual model, it results in a concave lien profile. This is opposite to that of a semi-amortizing loan that has a convex lien profile.



The net loan equation

$$Q_t = \int_t^T a e^{-rs} ds = \frac{a}{r} (e^{-rt} - e^{-rT}) \quad (29)$$

evaluated at  $t = 0$  gives<sup>12</sup>

$$Q_0 = \frac{a}{r} (1 - e^{-rT}) . \quad (30)$$

The maximal loan amount can thus be restated as follows

$$Q_0^{\max} = \frac{a_{\max}}{r} (1 - e^{-rT}) . \quad (31)$$

Imputing solutions for  $Q_0^{\max}$  (23) and  $a_{\max}$  (28), we can derive the *minimal loan tenure*  $T_{\min}$ .

We proceed from the following starting condition

$$P_0 e^{-\alpha \sigma \sqrt{t^*}} = \frac{\theta}{b} \frac{1}{r} (1 - e^{-rT}) . \quad (32)$$

By taking the natural logarithm of this function, the minimal loan tenure is given as

$$T \leq T_{\min} = -\frac{1}{r} \ln \left( 1 - \frac{br P_0 e^{-\alpha \sigma \sqrt{t^*}}}{\theta} \right) . \quad (33)$$

Thus, for  $T_{\min}$  to be a positive, *finite* real number, the logarithm must be a negative real, implying

$$0 < \frac{br P_0 e^{-\alpha \sigma \sqrt{t^*}}}{\theta} < 1 . \quad (34)$$

Note that:

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<sup>12</sup>This is analogous to  $Q_0 = A \gamma \frac{1-\gamma^T}{1-\gamma}$ .

1. If income capacity is infinite i.e.  $\theta = \infty$ , then equation (33) implies  $T_{\min} = 0$ . That is, the debt can be repaid instantly;
2. On the other hand, for the critical value of income,

$$\theta_{\min} = brP_0 e^{-\alpha\sigma\sqrt{t^*}}, \quad (35)$$

the debt must be perpetual, i.e.  $T_{\min} = \infty$ ; and

3. For any  $\theta < \theta_{\min}$  the entrepreneur is priced out i.e. has insufficient capacity to afford the debt.

We thus derived the optimal loan conditions (maximum initial debt  $Q_0^{\max}$ , maximum initial loan to value  $l_0^{\max}$ , minimum initial deposit  $ID_{\min}$  and the minimal loan tenure  $T_{\min}$ ) as a function of the borrower's (i.e., firm's) income  $\theta$ , underwriting asset value  $P_0$  and income constraints  $(\alpha, y, b)$ , exogenous interest rate  $r$ , along with asset characteristics  $(\mu, \sigma)$ .

For base case numbers  $\{\mu, r, \sigma, T\} = \{0.1, 0.05, 0.2, 30\}$ , and  $\alpha = 1, y = 99$  we obtained  $t^* = 20.8312$ . Now for  $b, P_0, \theta$  arbitrarily set at  $b = 5, P_0 = 350, \theta = 100$  we have

$$\frac{brP_0 e^{-\alpha\sigma\sqrt{t^*}}}{\theta} = \frac{5 * 0.05 * 350 * e^{-1*0.2\sqrt{20.8312}}}{100} = 0.35122 < 1 \quad (36)$$

$$T_{\min} = -\frac{1}{0.05} \ln(1 - 0.35122) \approx 8.7 \text{ yr} < \infty \text{ (repayment time is finite)} \quad (37)$$

$$\theta_{\min} = brP_0 e^{-\alpha\sigma\sqrt{t^*}} \approx 35 < \theta \text{ (income is sufficient)} \quad (38)$$

$$Q_0^{\max} = P_0 e^{-\alpha\sigma\sqrt{t^*}} \approx 140 \text{ (max loan} < \text{collateral} = 350) \quad (39)$$

$$l_0^{\max} \approx 40\% \text{ (LTV max)} \quad ID_{\min} \approx 210 \text{ (skin in game)} \quad (40)$$

In summary, meticulously structuring pragmatically default-free collateralized loan involves calibrating the loan endogenous parameters (initial deposit, loan repayment,

and tenure) contingent on the exogenous parameters (initial collateral value and its asset volatility, including the safety margin, discount factor, the borrower’s income capacity and its multiplier).

## 7 Conclusion

This paper discusses the vital issue of expunging agency costs of debt, comprising of both risk-shifting and underinvestment issues. We take a micro-economic stance to develop an algorithm involving option pricing technology. That is, we first start with a single project encompassing a risky real asset and illustrate the effacing of agency issues endemic in its underlying financial facility. Riskiness of the real asset is modelled via equation (6), which is the simplest but computationally tractable and well established way of modelling random increments of prices in continuous time.<sup>13</sup> Thus, once the option to default and the distress of underinvesting is obliterated for a single project firm, one can extend our same analysis to a firm with multiple projects — or, indeed, many firms with different projects in the economy, as in the KMV model<sup>14</sup> which also employs (correlated, but, in the limit, identical) GBM processes. This is the foundation model for capital requirement formulas of [Basel III \(2017\)](#), and formerly Basel II — despite the correlations between the projects. This is because the correlations merely change the payoff distributions without impacting on our option pricing methodology. If each firm’s debt is default-free or risk-free, correlations between the risk-free facilities of the firms in the economy do not matter. We can then use the principle of aggregation on the firms in a macro-economy to eradicate the menace of banking crisis. Our results in this sense are “overwhelming” (no pun intended).

Despite well established literature on the risk-reducing feature of collateral in loan

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<sup>13</sup>For the literature that asset prices follow a Geometric Brownian Motion (GBM) see [Hull \(2012\)](#).

<sup>14</sup>See [Vasicek \(1987\)](#) and [Vasicek \(1991\)](#).

contracts, the spate of loan defaults and repo run in the recent financial crisis underscores that mere presence of collateral does not fully dissipate default risk. The unravelling of sophisticated securitization and rehypothecation of debt instruments that emanates from poor loan practices emits system-wide financial fragility. Recent regulation is reactive rather than proactive in addressing the shortcomings of securitization (Foley, 2014). It also calls for steeper capital adequacy to bootstrap liquidity risk but with an attached economic cost (Hammond and Masters, 2013). We argue that post-crisis resolutions should be supplemented with upfront prudent underwriting practice, which is economically efficient to credit rehabilitation or bailout programs.

In this paper, we rationalize underwriting investable ‘safe’ assets. We contrast pragmatically default-free collateralized loans with default-prone risky loans. The former are characterized as ones that do not allow the put option to default to be significantly in-the-money, while the latter either have or will have their put options to default deep in-the-money. We illustrate that default-prone (risky) collateralized loans lead to potential disintermediation, i.e., negate intermediary’s core role of (i) asset transformation, and (ii) deposit custodian functions. Subsequent churning (securitization and rehypothecation) of a default-prone solutions, as exemplified by the repo run in the recent crisis, only transfers but not absolves the default risk to other market participants.

Our findings concur with Ebrahim and Hussain (2010) in that default-prone collateralized loan has a receding economic efficiency with heightened agency cost accruing to the risk of default. This is significant where the (i) recovery rate is stochastic with cram down actions by the borrower, or (ii) if recoveries from foreclosures are extremely low. In this situation, a pragmatically default-free loan is the only economically efficient solution. Even when agency costs are marginal, default-prone loan ranks at best economically neutral to its competing counterpart. This arises from the loan pricing that involves lender bearing the agency costs of debt. The above observations are distinct from the static and

dynamic trade off results of Myers (1984) and Strebulaev (2007) who aggregate the welfare of the opposing agents leading to erroneous result in poor states of economy.<sup>15</sup>

Pricing pragmatically default-free collateralized loans involves calibrating the endogenous loan parameters subject to its asset and income constraints that enable us to reduce the risk-shifting and underinvestment issues respectively. Our approach is consistent with that of Baltensperger (1978), Foote et al. (2008), Archer and Smith (2013) and Ebrahim (2009). This encompasses stripping the put option to default by ensuring the borrower's equity does not deteriorate into a negative region over all states of economy (i.e., for both types of asset with an upward or downward market trend). This strategy simultaneously sterilizes the feedback loop between collateral and credit cycles. By ensuring that borrower maintains adequate 'skin in the game,' it resolves the fragility and conflict of interest between financial intermediary and the borrower.

Our results provide policy implications in reinforcing the resilience of the present financial architecture. Regulators should emphasize the meticulous structuring of pragmatically default-free loans to mitigate financial fragility, in conjunction with granting firm's financial maneuverability or flexibility. This warrants: (a) sterilizing the put option to default by ensuring that borrowers maintain adequate 'skin in the game' (minimizing risk shifting tendencies); and (ii) evaluating an optimal tenure of the facility (minimizing underinvestment tendencies) over the duration of the loan. More importantly, constraining the put option to default endows depositors' similar security to a financial system with deposit protection scheme but without the moral hazard issues associated with such guarantees. Intuitively, this should prevent systemic crisis in a highly networked financial system and minimize cost on public funds arising from bailouts of failed financial intermediaries. Likewise, making allowance for affordability endows financial flexibility

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<sup>15</sup>Both studies aggregate the two adversarial claimants (debt and equity) objective functions, thereby depriving the analysis of supply and demand functions and hence the optimal pricing parameters of debt.

to firms. This allows them to undertake new projects thereby facilitating economic expansion. To sum up, the results of our study have the potential of not only reinforcing the resilience of the financial architecture but also that of promoting entrepreneurial activity and thus economic growth.

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## Appendix: Figures

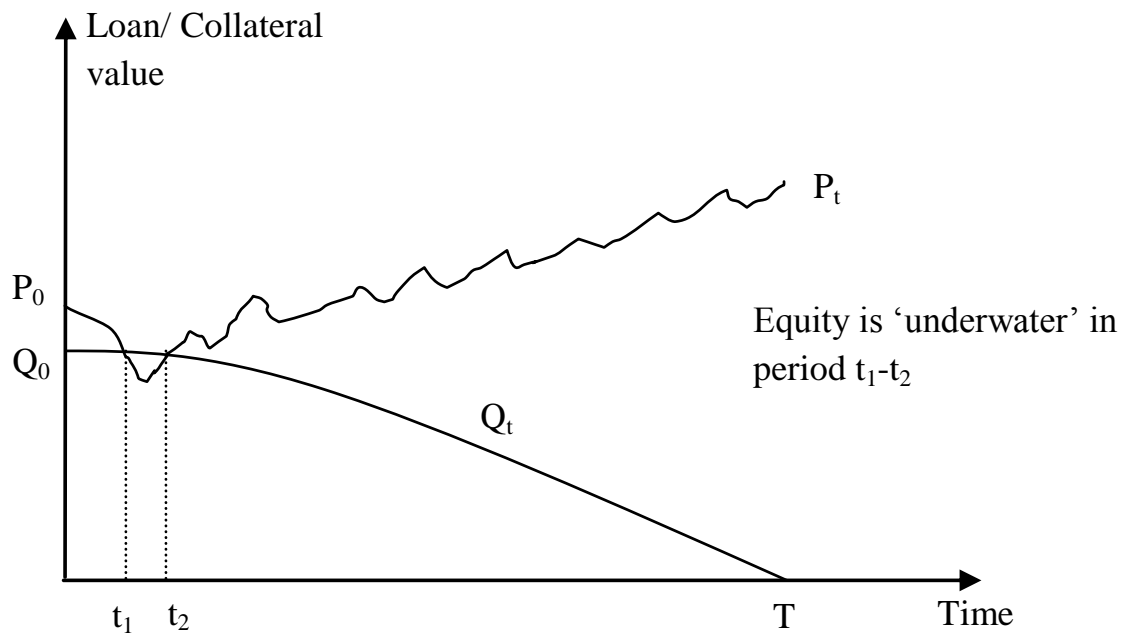


Figure 1: : **Risk-shifting/strategic default when equity goes underwater:** Default-prone collateralized loan.

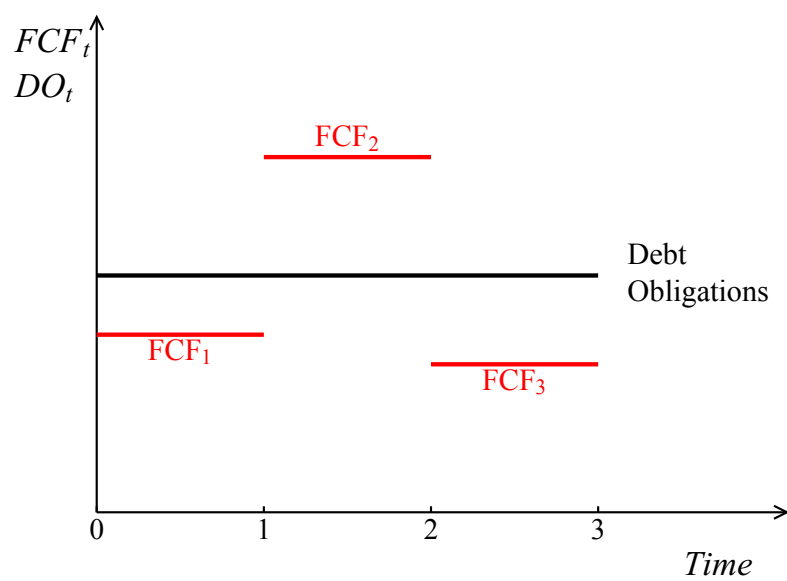


Figure 2: : Underinvestment when Free Cash Flows of borrower are below debt obligations.

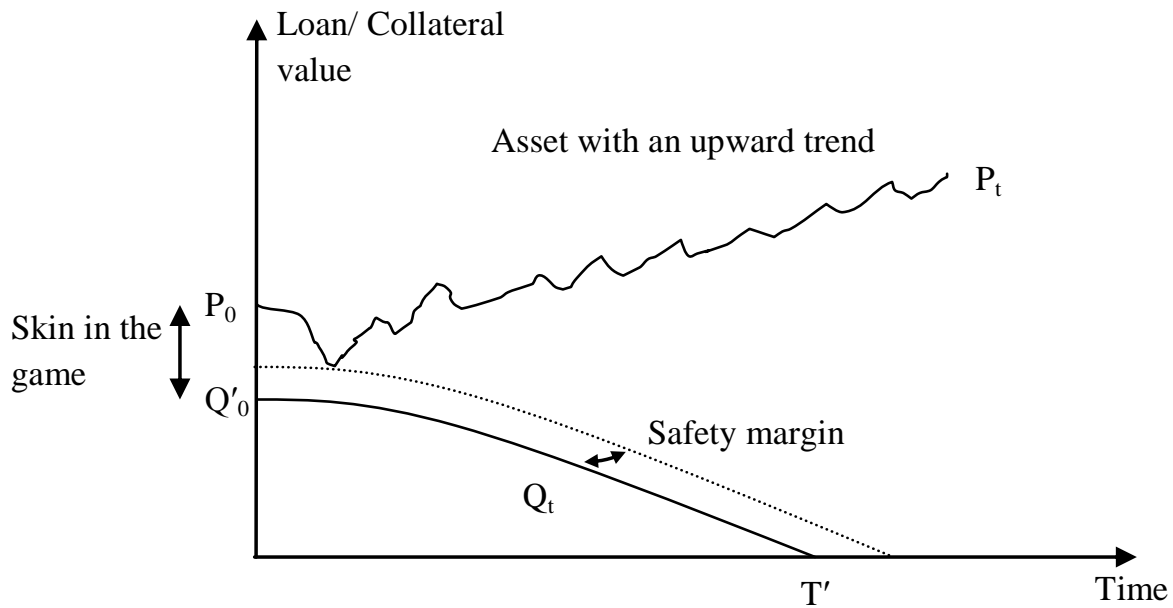


Figure 3: : Pragmatically default-free collateralized loan (with an upward trending asset).

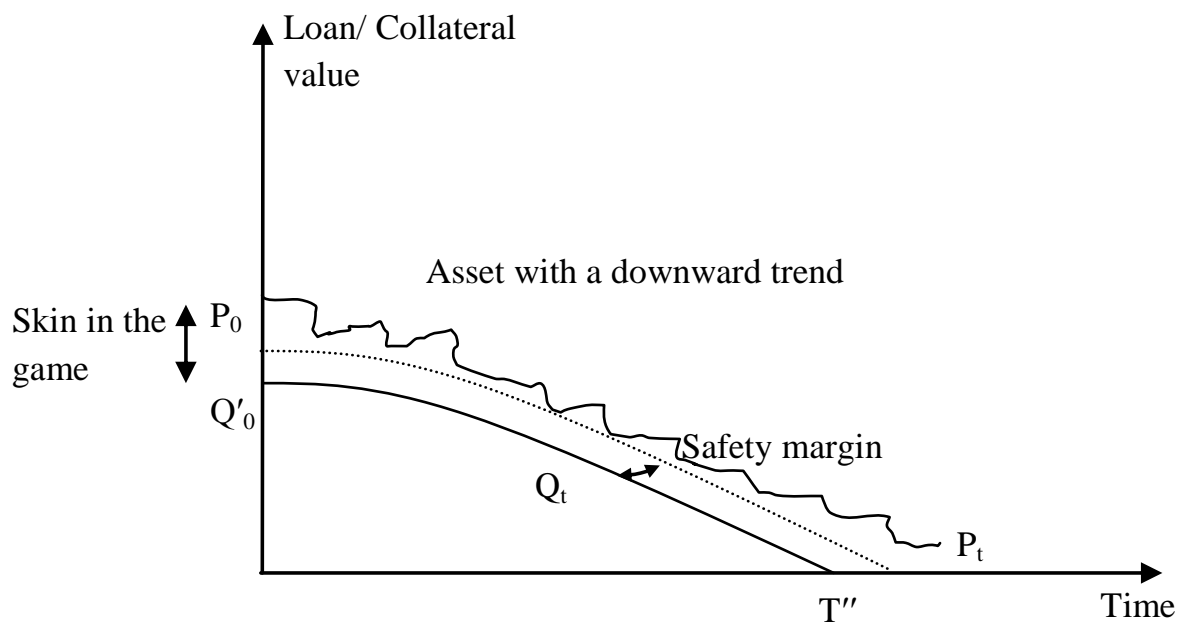


Figure 4: : Pragmatically default-free collateralized loan (with a downward trending asset).

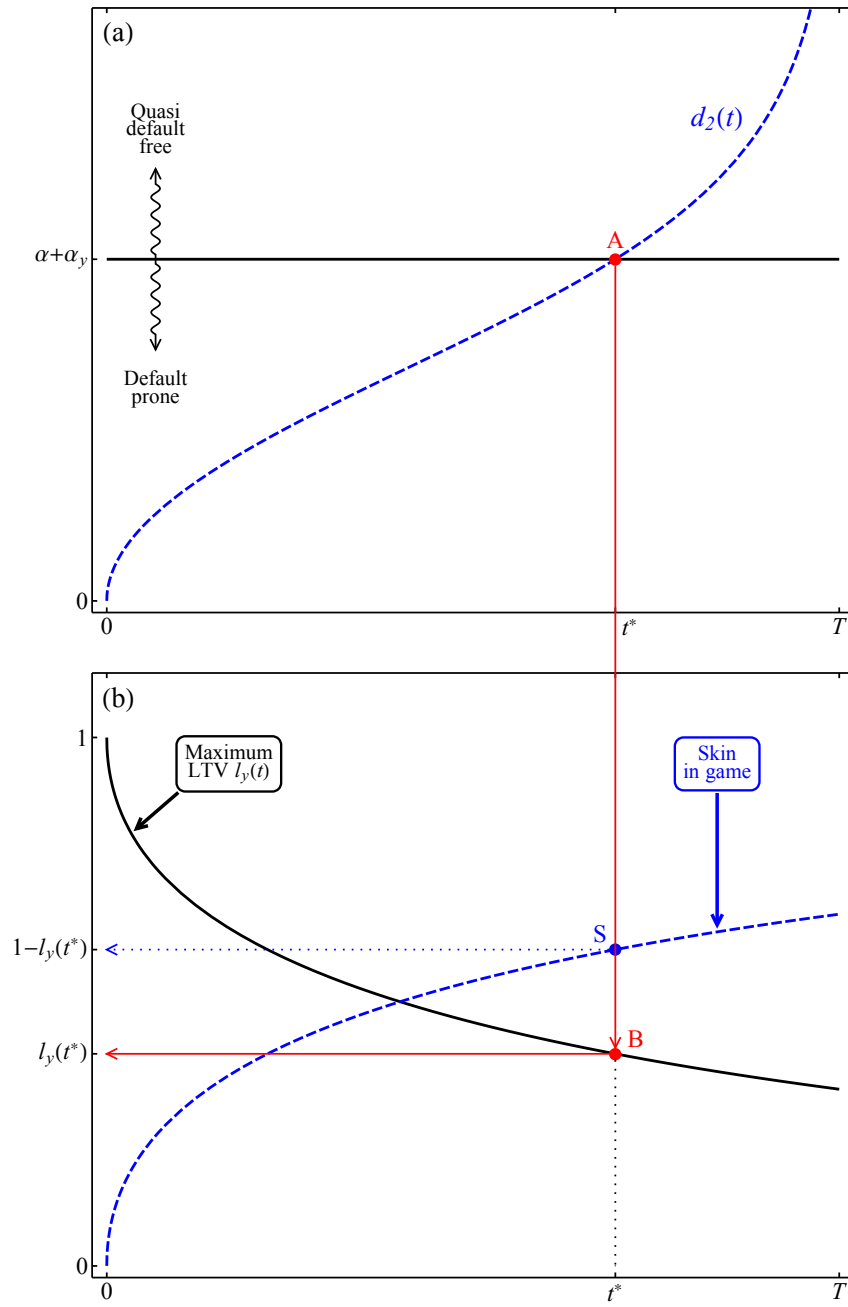


Figure 5: : **Optimal maximal loan to value ratio and minimal initial deposit fraction**  
 Solving numerically for critical  $t^*$  given sought security level  $y, \alpha_y$  gives the corresponding maximal loan to value ratio  $l_y(t^*)$  and the corresponding "skin in game" represented here as a percentage  $1 - l_y(t^*)$  of the initial project value  $P_0$ .